Beehive Designs for Observing Variety Competition

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Technical Report No. 186

Summary

Field plot layouts are devised for planting two varieties (+ and -) juxtaposed so as to achieve measurement of the yield of each under identical conditions of spatial competition with the other. Use is made of the equal distance to nearest neighbors and honeycombing properties of hexagonal forms. The replication of a triangular layout at 60° intervals around a central plot is used to achieve balanced symmetry and replication in mulitples of six. Yield can be measured on a plot surrounded by zero to six equally spaced competitors. A definition of "levelled" design is made to refer to the most nearly balanced number of replications of test hills per number of competitors.

Introduction

Field plot experiments designed to observe and evaluate variety competition have been used fairly extensively and particularly with soy beans. Hanson, et al., (1961) designed an experiment consisting of a random layout of parallel 16-foot single row plots with each variety defined as competing with its left and right neighbors. Schutz and Brim (1967) used both the parallel 16-foot single row plots and what we shall refer to as the nine hill plot placed on 3 x 3 grid lines. As an example:

B	B	B
B	A	B
B	B	B

represents the test variety A surrounded by the competitor B. The apparently successful use of nine hill plots in soy beans has lead to their use in evaluating competition in small grains as for example in Smith, et al., (1970) working with varieties of oats. The nine hill plot approach requires considerable experimental material and leads to difficulties in analysis because the corners are not equally distant from the side centers. The test variety can be surrounded with none, one, two, three or four competitors from the other variety by using successively all nine hills of A or one nearest neighbor of B, etc. The jump to five or more competitors does, however, involve a change in distance. The purpose of such experiments is to pit A against B and vice versa and observe the relation between mean yields and number of competitors, as Figure 1 suggests.

Figure 1 here

If, as Figure 1 is intended to suggest, both yields increase, there is cause for some interesting investigation. More probably, one relation will decrease but there may still be a point where a net gain in yield may be experienced by planting some ratio of one to the other. Figure 1 does not consider points between four and eight because of the mentioned difficulty, while in fact the location of eight on the scale is subject to question. No attempt is made here to suggest the nature of the relation, or to examine the unpleasant issue of accounting for within and between variety competitive correlations.

The author was asked to aid in designing an experiment for the purpose of constructing a picture similar to Figure 1. The problem originated in the College of Forestry at Minnesota and the experimental material consisted of a definitely limited supply of cuttings from distinct genetic backgrounds to be planted in flats in a greenhouse possessing the climate of a tranquilized hurricane and midwinter sunlight. Surrounding the test hill by six regularly spaced hills immediately cuts down on the material requirements and allows for the easy juxtaposition or intertwining of hills so that the material may serve multiple duty. For example:

```
  B B A
 B A B A
  B B A
```

yields test hill A with six competitor variety hills and test hill B with four competitor variety hills. Hexagonal plots are, of course, well known to researchers using field plots and were used by Mead (1967) to study spatial relations among hills of a given variety.

Beehive Designs

By using the honeycombing property of regular hexagons and the overlapping exhibited above, it seems possible to devise almost endless schemes for plot layout. A particular scheme which is well-structured, symmetrical and conveniently specified follows. Determine an equilateral triangle of side r-1 units long having r(r+1)/2 hills set out in the manner of bowling pins (referred to as the basic triangle, e.g. see Table 1). Extend the leading edge one unit further to a center, or axial, hill which shall be the same variety as its nearest neighbors. Now rotate the triangle layout at 60° intervals about the axial hill. Repreat the exact process on another hexagonal layout by interchanging the labels of the two varieties. A design consists of two hexagonal layouts of radius r units having 3r(r+1) + 1 hills each. Each hill in the basic triangle is replicated six times at 60° rotations in attitude and its interchanged reflection occurs in exactly the same manner. Each interior hill in the large plot is surrounded by six equally distant neighboring hills and serves as a test hill. Thus, ignoring the axial hill, the ratio of test to total hills planted is

$$\% \text{ test hills} = \frac{r(r-1)}{r(r+1)} \tag{1}$$

which approaches 1 as r gets larger. The percentage in (1) is larger than can be obtained using the nine hill rectangular scheme for the same amount of material. By designating one variety by + and the other by -, the first large hexagonal plot layout in a design becomes the negative of the second. The reader is again referred to the example in Table 1 before proceeding.

<u>Table 1 here</u>

There are 2^q, where $q = r(r+1)/2$, distinct designs of radius r, e.g. there are approximately two million designs of radius 6.

A test hill may be designated as being of the type surrounded by zero, one, two, three, four, five or six competitors and there are the same number of each type for both varieties. Let the vector $\underline{n} = (n_0, n_1, n_2, n_3, n_4, n_5, n_6)$ denote the number of replications of each type hill for each variety. Note that $n_i = 6k_i$ where k_i is the number of times a test hill with i competitors was determined in the basic triangle. Thus

$$\sum_{i=0}^{6} k_i = \frac{r(r-1)}{2} . \qquad (2)$$

Optimality and Existence

If we allow that the investigator has no knowledge of the possible complexity of the relation between a variety and the number of competitors, i, then we shall wish to spread the information as evenly as possible (maximizing entropy) over i.

Define

$$S(\min) = \min \sum_{i=0}^{6} k_i^2$$

where the minimization of the righthand side is subject to condition (2). If a design can be constructed such that $S = \sum_{i=0}^{6} k_i^2$ equals S(min), we say that the design is levelled. Tables 1 and 2 exhibit levelled designs for r = 4 and r = 5 respectively.

Designs of radius 6 have S(min) = 33. In Table 3, designs with S = 35 and S = 37 are exhibited. By the totally inelegant but manageable process of elimination of possibilities, this paper claims that no levelled designs of radius 6 exist.

The class of 2^{28} designs of radius 7 and the larger ones are being investigated at the time of this writing but their size seems impractical and the existence of levelled designs question becomes less manageable.

The author has not noticed a combinatorial system particularly relevant to this existence problem and welcomes suggestions or further work.

Table 2 here

Table 3 here

References

Hanson, W.D., Brim, C.A., and Hinson, K., "Design and Analysis of Competition Studies with an Application to Field Plot Competition in the Soybean," Crop Science, 1 (1961), 225-228.

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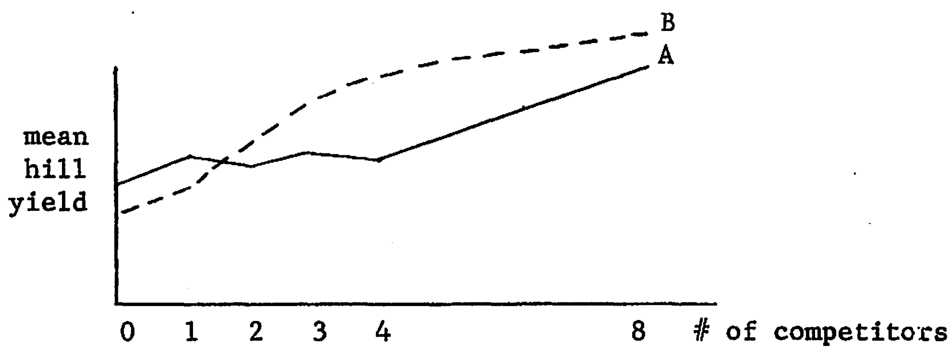

Figure 1. Competition yields

Table 1.

A Beehive Design of Radius 4.

Basic Triangle

```
+ + - +
 - + -
  + +
   +
```

```
    + + + - +
   - - - + - +
  + + + + + - +
 + - + + + + + -
+ - + + + + + - +
 - + + + + + - +
  + - + + + + +
   + - + - - -
    + - + + +
```

```
    - - - + -
   + + + - + -
  - - - - - + -
 - + - - - - - +
- + - - - - - + -
 + - - - - - + -
  - + - - - - -
   - + - + + +
    - + - - -
```

n = (6, 6, 6, 6, 6, 6, 0)
% testable hills = 60%

S = 6 = S(min)

Table 2

A Beehive Design of Radius 5.

Basic Triangle

```
- + - - -
 - + + +
  + - +
   + +
    +
```

$n = (6, 12, 12, 12, 6, 6, 6)$ $S = 16 = S(\min)$
% testable hills = 67%

Table 3

Some Beehive Designs of Radius 6.

Basic Triangle

```
+ + - - - -
 + - + - +
  + - + +
   + + -
    + +
     +
```

n = (12, 12, 18, 18, 12, 12, 6)   S = 35
% testable hills = 71%   S(min) = 33

Basic Triangle

```
+ + - + + +
 + - + - +
  + - + +
   + + -
    + +
     +
```

n = (12, 18, 18, 12, 12, 6, 12)   S = 35

Basic Triangle

```
- - + + - -
 - + + - +
  + - + -
   + + -
    + +
     +
```

n = (12, 6, 18, 18, 18, 6, 12)   S = 37